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Het eerstvolgende NIOC vindt plaats in 2027 en wordt dan georganiseerd door HAN University of Applied Sciences. Zodra daarover meer informatie beschikbaar is, is deze hier te vinden.

Wil je op de hoogte blijven van de ontwikkeling rond Stichting NIOC en de NIOC kennisbank, schrijf je dan in op de nieuwsbrief via

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MathDox

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Abstract
MathDox is a system for presenting highly interactive mathematical documents over the World Wide Web. It provides easy connections to Computer Algebra Systems, dynamic geometry systems and other mathematical services. Within MathDox we have developed an exercise system providing parameterized exercises to students and offering intelligent feedback. We will describe the MathDox-system, its applications and use in concrete situations.

Key Words
interactive mathematics, mathdox, exercise system

MathDox

Interactive Mathematics in education

1 Introduction

Recently developed E-learning tools like Learning Management Systems (LMS) are very well suited for distance education and for delivery of online educational materials to students. However, these systems only offer a good learning environment for eduction via the web. The success of good education is still very dependent on the learning material itself.

In the area of Mathematics we see various new tools offering students a rich environment for practicing mathematics. These systems offer parameterized exercises with open questions that are automatically graded. Think of Maple TA, Aleks, and WebAssign, or STACK, WIMS, WebWORK, DWO and ActiveMath. Here we report on our own initiatives to create a system, called **MathDox**, offering highly interactive mathematical documents and exercises.

MathDox is an open source system serving dynamic and interactive web pages with high quality rendering of mathematics and easy access to mathematical services like computer algebra systems or dynamic geometry software. The power of MathDox is that it provides users with various means to communicate with mathematical services and offers a flexible way for providing intelligent feedback in its documents and exercise system.
The MathDox software is open source and can be found at www.mathdox.org.

2 Interactive mathematical documents

Web pages about mathematics are often static and do not invite the user to experiment with and experience the material. We see three main reasons for this.
First of all, interactivity in mathematical content often requires non-trivial computations that are not easily performed without the assistance of specialized mathematical software, e.g. computer algebra systems.
Second, displaying mathematics properly on a web page can be challenging.
Third, the input of mathematical expressions is difficult, both for the user as well as for the author.
To address these issues we have developed the MathDox system, see (Cohen et al, 2006) and (Cuypers et al., 2008). It consists of the MathDox format and a set of tools including the MathDox Player. The MathDox format is an XML based language for interactive mathematical documents. As such MathDox documents can be interpreted and transformed into web pages by the MathDox Player. The resulting web pages are dynamic and interactive, support rendering of mathematics, offer easy access to computer algebra systems, and are equipped with a convenient mathematical input system.
We describe the various parts of the system.

3 The MathDox Format

MathDox sources combine in a modular way various existing XML formats. Each format contributes a useful facet for interactive documents. The main XML formats used in MathDox are:

– DocBook, for structuring documents
– OpenMath, for semantic encoding of mathematics.
– Xforms
– Jelly, a programming and scripting format.

These formats, their use and purpose, will be discussed.

3.1 DOCBOOK

DocBook is a well-known documentation standard, see (Walsh, 1999). Its logical structure facilitates searching and parsing of specific elements, enabling easy translation into other formats including HTML and PDF.

3.2 OPENMATH

Within MathDox mathematics appears in various forms: Mathematics as used in computations by software packages or mathematics solely meant for the user to read or sometimes for both the computer and the user. For each type of usage one wants mathematics to have specific properties. In general a user will be able to grasp the meaning behind a, possibly ambiguous, mathematical expression. Computer software, however, will need its mathematics to be completely unambiguous, since it cannot benefit from the context in the way a mathematically skilled reader does in order to solve gaps caused by incompleteness and ambiguity.

OpenMath, see (Buswell et al, 2004) has been chosen as the main format for mathematics within MathDox documents. It is well suited because it is semantically rich, unambiguous, XML-based and can easily be transformed into other formats, such as MathML and LaTeX, which are better suited for presentation. These transformations allow MathDox documents to use OpenMath where semantics are important and switch to MathML or LaTeX, when the mathematics need to be presented to the user either on screen or on paper.

3.3 XFORMS

XForms (http://www.w3.org/TR/xforms11/) in MathDox documents supplies a means of interaction with the reader of the MathDox document. It allows for many interaction (form) elements which can be used for user input, such as text fields (also formulas), radio buttons, and drop down menus. In MathDox the XForms elements are used to trigger interaction with mathematical services like computer algebra systems.

3.4 JELLY

To specify and fine-tune reactions of a MathDox document to user input, an author needs programming constructs. For this purpose Jelly (http://commons.apache.org/jelly/) has been included in the MathDox format. Jelly is a JSP-like XML-language (http://java.sun.com/products/jsp/), and has been developed as part of the Apache project (http://www.apache.org/). Jelly can be used for conditional statements, loops, variables, calls to Java (http://java.sun.com/) objects and calls to web services.

4 MathDox software

So far we only discussed the MathDox format and just briefly mentioned the server-side software responsible for processing of this format. In this section we discuss the MathDox software.
The **MathDox Player** is the main component of our software. It is responsible for making MathDox documents accessible over the web. Its task can be compared with that of a web server, in the sense that both a web server and the MathDox Player offer stored documents from the server to the outside world. The main difference between a normal web server and the MathDox Player is that a web server offers ready-made HTML files, whereas the MathDox Player dynamically creates these HTML files. On request of the user the MathDox server collects data from the source document, the user and from mathematical back engines providing services and creates, by applying a number of XSL transformations on this input, a new view on the document.

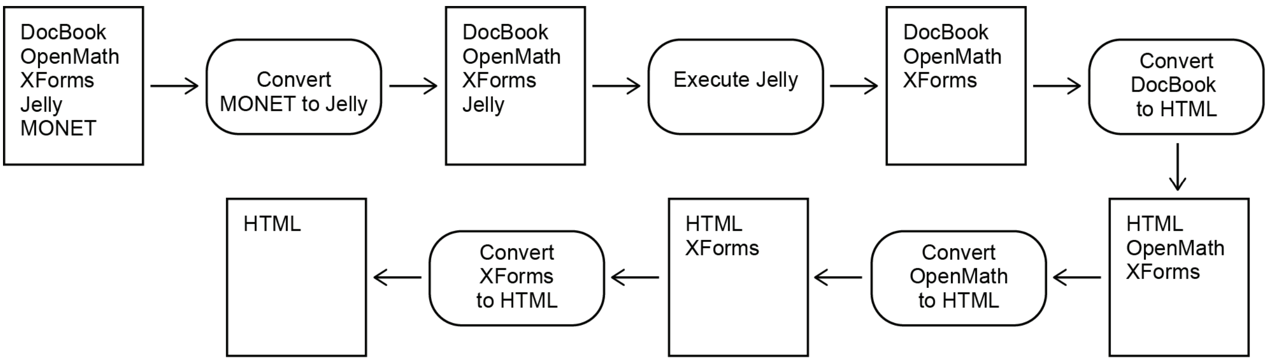

FIGURE 1 A schematic overview of the pipeline of operations carried out by the MathDox player

The user interface to the MathDox player is a modern web browser. Interaction of the user is realized by the various options that HTML offers. However, there is no standard way to communicate mathematics in a meaningful way. Especially since we want to enable the interaction with various mathematical services offered by the system, we require the user to provide semantically rich mathematical expressions. To this end we have developed a formula editor, see www.mathdox.org/formulaeditor/.

The **MathDox formula editor** features:
– a two-dimensional WYSIWYM (what you see, is what you mean) interface
– semantic representation of the formulas in OpenMath.
The editor is written in Javascript and can easily be integrated in MathDox and HTML web pages.

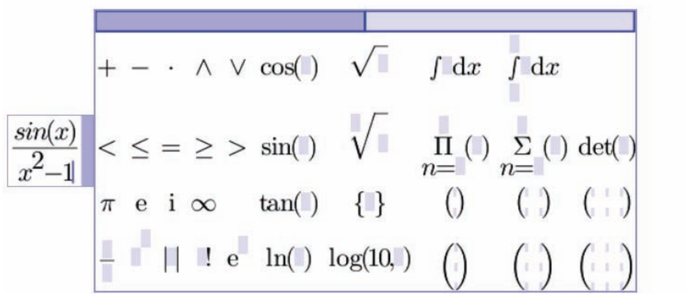

FIGURE 2 The MathDox Formula Editor

5 The MathDox exercise system

One of the main applications of MathDox is the **MathDox exercise system.**
Basically there are two types of exercises possible within the exercise system: multiple choice and open questions and all kinds of combinations thereof.
Within MathDox it is possible, with the use of a computer algebra system to compare the answers given by the user to that of the author of the exercise. The computer algebra system will also eliminate the problem that mathematical expressions can be written in many different ways while the semantic meaning remains the same. This allows for more complex answers and variations of the same answer as entered by the author.
Another advantage of MathDox is the possibility of randomising values used in the question and expected answer.
MathDox also supports multiple step exercises. These exercises can be designed to capture mistakes and address them or correct the understanding of the user. Feedback provided in the exercise may vary from a hint, a reference to the theory, or by simplifying the question.
A MathDox exercise can be considered to be an automaton. The various states of the automaton, called *interactions*, are presented by the MathDox player as separate web pages to the user. These interactions may contain the question of an exercise, the feedback on a correct or wrong answer, or some hints. The transitions from one state into another, called *answer_map* are ruled by the actions of the user, and the results of various *queries* to mathematical services available within the exercise. We explain this by way of the following example:

Determine $\frac{1}{4}+\frac{1}{6}$.

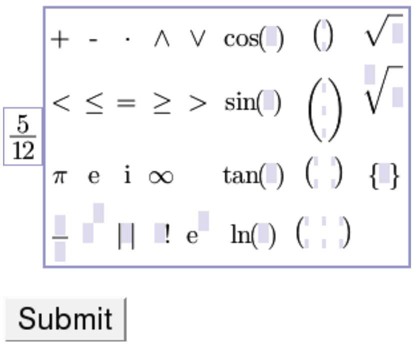

FIGURE 3 Illustration 1: Adding fractions

This is the question of an exercise, it is the first state of the automaton that the user will encounter when starting up the exercise. Now the user may provide an answer, correct or incorrect. For each of these two possibilities the exercise contains an interaction, that the user is redirected to after supplying his answer. The XML-encoding of this exercise is given below.

```
<exercise>
  <interaction id=”question”>
    <para>Determine
      <OMOBJ>
        <OMA><OMS name=”plus” cd=”arith1”/>
          <OMA><OMS name=”divide” cd=”arith1”/>
            <OMI>1</OMI><OMI>4</OMI>
          </OMA>
          <OMA><OMS name=”divide” cd=”arith1”/>
```

```
            <OMI>1</OMI><OMI>6</OMI>
          </OMA>
        </OMA>
          </OMOBJ>
    </para>
    <para>
      <userinput type="blank"><set name="answer/></userinput>
    </para>
      <answer_map>
        <choose>
          <when target="correct">
            <query>
              <OMOBJ>
                <OMA><OMS name="eq" cd="relation1"/>
                  <out name="answer"/>
                <OMA><OMS name="divide" cd="arith1"/>
                  <OMI>5</OMI><OMI>12</OMI>
                </OMA>
              </OMA>
            </OMOBJ>
          </query>
        </when>
      <otherwise target="incorrect"/>
    </choose>
  </answer_map>
</interaction>
<interaction id="correct"><para>Well  done! </para></interaction>
<interaction id="incorrect">
  <para> Sorry, that is wrong.</para>
<answer_map text="Try again">
  <interaction xref="question"/>
</answer_map>
</interaction>
</exercise>
```

The question is posed in the top interaction with *id* equal to "question". In that interaction one also finds a *userinput*-tag. This tag represents an input field for the user. Inside the web page presenting this interaction the userinput-tag is replaced by the MathDox-formula editor, see below. The answer that the user can submit via this formula-editor is set to a variable with name the "answer". In the *answer_map* at the end of the interaction, a *query* (to, for example, a CAS) is defined that checks whether the answer is equal to 5/12. This answer_map redirects the user either to the interaction with id "correct" in case the query returns a "true", or the interaction with id "incorrect" otherwise. The latter interaction offers the possibility to try the exercise again.

The combination of queries to a CAS and applications of our Jelly tools makes it possible to do the following operations on OpenMath expressions:
– check expressions to be equal
– check an expression to be semantically equivalent to a given one;
– analyze the syntax of an expression;
– extract subexpressions of an answer;
– check an expression on the occurrence or absence of a symbol.
Indeed, all the checks described by (Sangwin, 2007) can be easily implemented, as well as many more.

6 MathDox at work

The MathDox software offers various ways to interact with mathematical documents. To combine the strength of MathDox with the facilities of an e-learning environment, we extended the MathDox exercise system with a scoring mechanism compatible with the SCORM-standard. This makes it possible to include MathDox pages and exercises in SCORM-packages and upload them in e-learning environments like Blackboard or Moodle.

At Eindhoven University of Technology we have set up various e-learning sites based on the MathDox software embedded in a Moodle e-learning environment:

– **Wortel TU/e**; this is the open mathematical e-learning site of the Eindhoven University of Technolgy. Here one finds course material for various (undergraduate) courses at first year university level and for high school students. The site can be found at wortel.tue.nl.

– **Telmme**; for the past couple of years there is a substantial increase of the number of students entering a masters program at one of the three Dutch universities of technology who obtained their bachelor degree at another university or in an other country. One of the effects is that the group of students entering a masters program is very diverse, not in the least with respect to their mathematical background. To give students the possibilities to test their mathematical knowledge and require the necessary mathematical skills, the three Dutch universities of technology offer online study material on mathematics via the e-learning site www.telmme.nl.

– **WisTU/e**; on entering the Eindhoven University of Technology, all new students do take a test on mathematics. To prepare them for this test, they can practice at www.wistue.nl.

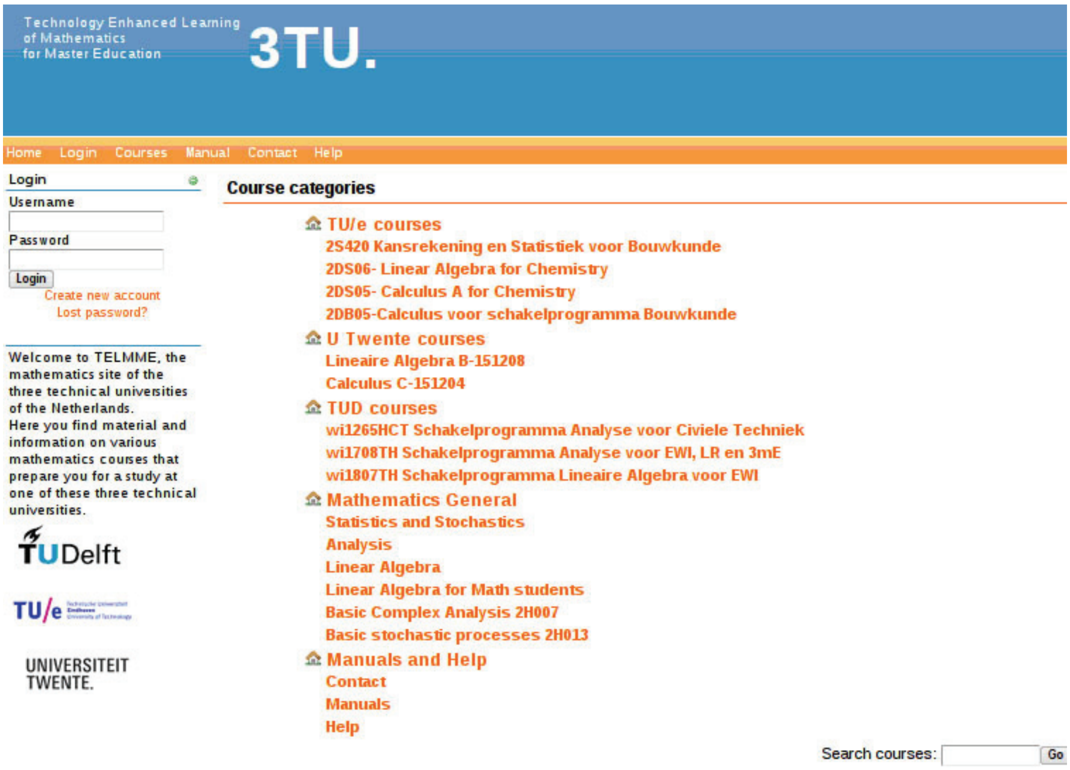

FIGURE 4 The Telmme e-learning environment

From the student and teacher evaluations of the described systems it becomes clear that both groups really appreciate the extra possibilities that MathDox offers. In particular, the immediate and relevant feedback on exercises is highly appreciated. However, both students and teachers do not want to see the e-learning as offered by the MathDox environment as a substitution for the classical way of teaching, but they consider it to be valuable addition.

A thorough investigation on the effects of the intelligent feedback as provided by the MathDox system in the area of linear algebra has been carried out in (Corbalan et al., 2010). This research has shown that there is a significant improvement of the far transfer of knowledge, meaning that students hat have studied the material with the help of more intelligent feedback do have more insight in more complex situations and exercises.

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